General covariance and its implications for Einstein's space-times.

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Abstract

This is a review of the chrono-geometrical structure of special and general relativity with a special emphasis on the role of non-inertial frames and of the conventions for the synchronization of distant clocks. ADM canonical metric and tetrad gravity are analyzed in a class of space-times suitable to incorporate particle physics by using Dirac theory of constraints, which allows to arrive at a separation of the genuine degrees of freedom of the gravitational field, the Dirac observables describing generalized tidal effects, from its gauge variables, describing generalized inertial effects. A background-independent formulation (the rest-frame instant form of tetrad gravity) emerges, since the chosen boundary conditions at spatial infinity imply the existence of an asymptotic flat metric. By switching off the Newton constant in presence of matter this description deparametrizes to the rest-frame instant form for such matter in the framework of parametrized Minkowski theories. The problem of the objectivity of the space-time point-events, implied by Einstein's Hole Argument, is analyzed.

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I. INTRODUCTION

I will illustrate the status of the understanding of the chrono-geometrical structure of special and general relativity from the Hamiltonian point of view. Then I will speak about Einstein's Hole Argument and of our understanding of the objectivity of space-time point-events. Finally I will present a biased list of open problems in this area.

Instead of merging in an enumeration of the main hot problems of contemporary research like black holes and their entropy, cosmic censorship and singularity theorems, string and M-theory, loop quantum gravity and quantum geometry, rotating stars in astrophysics and gravitational collapse, dark energy, dark matter and the anisotropy of the cosmic background radiation in the cosmological context, gravitational lensing, gravitational waves and their detection, tests on gravity theories from solar system experiments and binary stars, I will review the basic notions of relativity emphasizing aspects like the lack of a simultaneity notion and the importance of a formulation able to give a well-posed Cauchy problem. Due to general covariance Einstein's equations are a system of ten partial differential equations, which cannot be put in normal form. Four of them are not independent from the others due to Bianchi identities. Other four are only restrictions on initial data. As a consequence only two of them contain a genuine dynamical information and eight components of the 4-metric tensor are left undetermined by Einstein's equations. Therefore the formulation of their Cauchy problem is extremely complicated [see Rendall (1998) and Friedrich and Rendall (2000)for a modern assessment]. However this problem can be attached in a systematic way in the Hamiltonian approach based on Dirac theory of constraints and the associated canonical formulation of the second Noether theorem [see Lusanna (1993)]. In this way it is possible to develop a strategy for the determination of the genuine degrees of freedom of the gravitational field (the Dirac observables, denoted as DO in what follows), describing its tidal effects, and of a set of hyperbolic Hamilton equations for their evolution after having fixed all the arbitrary gauge variables of the gravitational field, describing its inertial effects. Therefore, a preliminary problem is to find a well-posed description of special relativistic systems in non-inertial frames starting from the standard one in inertial frames dictated by the relativity principle.

This presentation is based on a theoretical physics viewpoint aiming to unify gravity and particle physics and to understand how to reconcile general relativity with quantum theory. I hope to be able to give a feeling of how *heuristic* are most of the results due to the lack of sufficient mathematical rigor of many of the tools needed to treat these problems and to stimulate mathematicians to develop new ideas to refine them.

II. THE CHRONOGEOMETRICAL STRUCTURE OF SPECIAL RELATIVITY

In the *Annus Mirabilis* 1905 Einstein was able to reconcile the relativity principle with Maxwell electrodynamics incorporating Lorentz's partial results and eliminating the concept of aether. See Norton (2005) for a suggestive reconstruction of Einstein's line of reasoning to achieve this result. The outcome was the elimination of Newton absolute time and of the absolute Euclidean 3-space associated to each instant of time. According to Newton this instantaneous 3-space had to be interpreted as a container for matter and its existence amounts to the philosophical *substantivalist* position. This viewpoint was never accepted by

Leibniz, whose *relationist* position refuses the notion of an absolute location of bodies: they are only defined by their mutual relations with other bodies. The notion of a container of matter was put in crisis by the advent of Maxwell electrodynamics, in which fields pervaded the whole universe, while the relationist point of view influenced Einstein through the ideas of Mach.

The Galilei group connecting the non-relativistic inertial frames in accord with the non-relativistic relativity principle was replaced with the Poincare' group (containing Lorentz transformations as a subgroup) connecting the relativistic ones inside the absolute Minkowski space-time according to the relativistic relativity principle. In both cases Cartesian coordinates were privileged by this principle. Moreover the two-way (or round-trip) velocity of light (only one clock is needed in its definition) was assumed to be c, namely *constant* and *isotropic* (the light postulates), by Einstein. The resulting time dilatations and length contraction under Lorentz transformations became kinematical notions, contrary to Lorentz's viewpoint according to which they were physical phenomena, while the light postulates were only a kinematical convention.

Therefore Einstein's revolution led to unify the independent notions of time and space in the notion of a 4-dimensional manifold, the Minkowski space-time, with an absolute (namely non-dynamical) chrono-geometrical structure. The Lorentz signature of its 4-metric tensor implies that every time-like observer can identify the light-cone (the conformal structure, i.e. the locus of the trajectories of light rays) in each point of the world-line. But there is *no notion of an instantaneous 3-space, of a spatial distance and of a one-way velocity of light between two observers* (the problem of the synchronization of distant clocks). Since the relativity principle privileges inertial observers and Cartesian coordinates $x^\mu = (x^o = ct; \vec{x})$ with the time axis centered on them (inertial frames), the $x^o = const.$ hyper-planes of inertial frames are usually taken as Euclidean instantaneous 3-spaces, on which all the clocks are synchronized. Indeed they can be selected with Einstein's convention for the synchronization of distant clocks to the clock of an inertial observer. This inertial observer A sends a ray of light at x^o_i to a second accelerated observer B, who reflects it towards A. The reflected ray is reabsorbed by the inertial observer at x^o_f. The convention states that the clock of B at the reflection point must be synchronized with the clock of A when it signs $\frac{1}{2}\,(x^o_i + x^o_f)$. This convention selects the $x^o = const.$ hyper-planes of inertial frames as simultaneity 3-spaces and implies that with this synchronization the two-way and one-way velocities of light coincide and the spatial distance between two simultaneous point is the (3-geodesic) Euclidean distance.

However, real observers are never inertial and for them Einstein's convention for the synchronization of clocks is not able to identify globally defined simultaneity 3-surfaces, which could also be used as Cauchy surfaces for Maxwell equations. The 1+3 *point of view* tries to solve this problem starting from the local properties of an accelerated observer, whose world-line is assumed to be the time axis of some frame. Since only the observer 4-velocity is given, this only allows to identify the tangent plane of the vectors orthogonal to this 4-velocity in each point of the world-line. Then, both in special and general relativity, this tangent plane is identified with an instantaneous 3-space and 3-geodesic Fermi coordinates are defined on it and used to define a notion of spatial distance. However this construction leads to coordinate singularities, because the tangent planes in different points of the world-line will intersect each other at distances from the world-line of the order of the (linear and rotational) *acceleration radii* of the observer (see Mashhoon and Muench (2002) for their definition). Another type of coordinate singularity arises in all the proposed uniformly rotating coordinate systems: if ω is the constant angular velocity, then at a distance r from the rotation axis such that $\omega\, r = c$, the ${}^4g_{oo}$ component of the induced 4-metric vanishes. This is the so-called *horizon problem for the rotating disk*: the time-like 4-velocity of an observer sitting on a point of the disk becomes light-like in this coordinate system when $\omega\, r = c$.

While in particle mechanics one can avoid these problem and formulate a theory of measurement based on the *locality hypothesis* [standard clocks and rods do not feel acceleration and at each instant the detectors of the instantaneously comoving inertial observer give the correct data; see Mashhoon (1990, 2003)], this methodology does not work with continuous media (for instance the constitutive equations of the electromagnetic field inside them in non-inertial frames are unknown) and in presence of electromagnetic fields when their wavelength is comparable with the acceleration radii of the observer (the observer is not enough "static" to be able to measure the frequency of such a wave).

See Alba and Lusanna (2003) for a review of these topics.

This state of affairs and the need of predictability (a well-posed Cauchy problem for field theory) lead to the necessity of abandoning the 1+3 point of view and to shift to the 3+1 one. In this point of view, besides the world-line of an arbitrary time-like observer, it is given a 3+1 splitting of Minkowski space-time, namely a foliation of it whose leaves are space-like hyper-surfaces. Each leaf is both a Cauchy surface for the description of physical systems and an instantaneous (in general Riemannian) 3-space, namely a notion of simultaneity implied by a clock synchronization convention different from Einstein's one. Even if it is unphysical to give initial data on a non-compact space-like hyper-surface, this is the only way to be able to use the existence and uniqueness theorem for the solutions of partial differential equations. In the more realistic mixed problem, in which we give initial data on the Earth and we add an arbitrary information on the null boundary of the future causal domain of the Earth (that is we prescribe the data arriving from the rest of the universe, the ones observed by astronomers), the theorem cannot be shown to hold!

The extra structure of the 3+1 splitting of Minkowski space-time allows to enlarge its atlas of 4-coordinate systems with the definition of *Lorentz-scalar observer-dependent radar 4-coordinates* $\sigma^A = (\tau; \sigma^r)$, $A = \tau, r$. Here τ is either the proper time of the accelerated observer or any monotonically increasing function of it, and is used to label the simultaneity leaves Σ_τ of the foliation. On each leaf Σ_τ the point of intersection with the world-line of the accelerated observer is taken as the origin of curvilinear 3-coordinates σ^r, which can be assumed to be globally defined since each Σ_τ is diffeomorphic to R^3. To the coordinate transformation $x^\mu \mapsto \sigma^A$ (x^μ are the standard Cartesian coordinates) is associated an inverse transformation $\sigma^A \mapsto x^\mu = z^\mu(\tau, \sigma^r)$, where the functions $z^\mu(\tau, \sigma^r)$ describe the embedding of the simultaneity surfaces Σ_τ into Minkowski space-time. The 3+1 splitting leads to the following induced 4-metric (a functional of the embedding): ${}^4g_{AB}(\tau, \sigma^r) = \frac{\partial z^\mu(\sigma)}{\partial \sigma^A}\, {}^4\eta_{\mu\nu}\, \frac{\partial z^\nu(\sigma)}{\partial \sigma^B} = {}^4g_{AB}[z(\sigma)]$, where ${}^4\eta_{\mu\nu} = \epsilon\,(+---)$ with $\epsilon = \pm 1$ according to particle physics or general relativity convention respectively. The quantities $z^\mu_A(\sigma) = \frac{\partial z^\mu(\sigma)}{\partial \sigma^A}$ are cotetrad fields on Minkowski space-time.

An admissible 3+1 splitting of Minkowski space-time must have the embeddings $z^\mu(\tau, \sigma^r)$

of the space-like leaves Σ_τ of the associated foliation satisfying the Møller conditions on the coordinate transformation [see Møller (1957)]

$$\epsilon\, {}^4g_{\tau\tau}(\sigma) > 0,$$

$$\epsilon\, {}^4g_{rr}(\sigma) < 0, \qquad \begin{vmatrix} {}^4g_{rr}(\sigma) & {}^4g_{rs}(\sigma) \\ {}^4g_{sr}(\sigma) & {}^4g_{ss}(\sigma) \end{vmatrix} > 0, \qquad \epsilon\, det\, [{}^4g_{rs}(\sigma)] < 0,$$

$$\Rightarrow det\, [{}^4g_{AB}(\sigma)] < 0.$$

Moreover, the requirement that the foliation be well defined at spatial infinity may be satisfied by asking that each simultaneity surface Σ_τ tends to a space-like hyper-plane there, namely we must have $z^\mu(\tau, \sigma^r) \rightarrow x^\mu(0) + \epsilon^\mu_A\, \sigma^A$ for some set of orthonormal asymptotic tetrads ϵ^μ_A.

As a consequence, any admissible 3+1 splitting leads to the definition of a *non-inertial frame centered on the given time-like observer* and coordinatized with Lorentz-scalar observer-dependent radar 4-coordinates. While inertial frames centered on inertial observers are connected by the transformations of the Poincare' group, the non-inertial ones are connected by passive frame-preserving diffeomorphism: $\tau \mapsto \tau^{'}(\tau, \sigma^r)$, $\sigma^r \mapsto \sigma^{'\,r}(\sigma^s)$. It turns out that Møller conditions forbid uniformly rotating non-inertial frames: only differentially rotating ones are allowed (the ones used by astrophysicists in the modern description of rotating stars). In Alba and Lusanna (2005a) there is a detailed discussion of this topic and there is the simplest example of 3+1 splittings whose leaves are space-like hyper-planes carrying admissible differentially rotating 3-coordinates. Moreover, it is shown that to each admissible 3+1 splitting are associated two congruences of time-like observers (the natural ones for the given notion of simultaneity): i) the Eulerian observers, whose unit 4-velocity field is the field of unit normals to the simultaneity surfaces Σ_τ; ii) the observers whose unit 4-velocity field is proportional to the evolution vector field of components $\partial z^\mu(\tau, \sigma^r)/\partial\tau$: in general this congruence is non-surface forming having a non-vanishing vorticity (like the congruence associated to a rotating disk).

The next problem is how to describe physical systems in non-inertial frames and how to connect different conventions for clock synchronization. The answer is given by *parametrized Minkowski theories* [see Lusanna (1997, 2004)]. Given any isolated system (particles, strings, fields, fluids) admitting a Lagrangian description, one makes the coupling of the system to an external gravitational field and then replaces the 4-metric ${}^4g_{\mu\nu}(x)$ with the induced metric ${}^4g_{AB}[z(\tau, \sigma^r)]$ associated to an arbitrary admissible 3+1 splitting. The Lagrangian now depends not only on the matter configurational variables but also on the embedding variables $z^\mu(\tau, \sigma^r)$ (whose conjugate canonical momenta are denoted $\rho_\mu(\tau, \sigma^r)$). Since the action principle turns out to be invariant under frame-preserving diffeomorphisms, at the Hamiltonian level there are four first-class constraints $\mathcal{H}_\mu(\tau, \sigma^r) = \rho_\mu(\tau, \sigma^r) - l_\mu(\tau, \sigma^r)\, T^{\tau\tau}(\tau, \sigma^r) - z^\mu_s(\tau, \sigma^r)\, T^{\tau s}(\tau, \sigma^r) \approx 0$ in strong involution with respect to Poisson brackets, $\{\mathcal{H}_\mu(\tau, \sigma^r), \mathcal{H}_\nu(\tau, \sigma^r_1)\} = 0$. Here $l_\mu(\tau, \sigma^r)$ are the covariant components of the unit normal to Σ_τ, while $z^\mu_s(\tau, \sigma^r)$ are the components of three independent vectors tangent to Σ_τ. The quantities $T^{\tau\tau}$ and $T^{\tau s}$ are the components of the energy-momentum tensor of the matter inside Σ_τ describing its energy- and momentum-densities. As a consequence, Dirac's theory of constraints (or its geometrical version as presymplectic geometry when only first-class constraints are present) implies that the configuration variables $z^\mu(\tau, \sigma^r)$ are arbitrary *gauge variables*. Therefore, all the admissible 3+1 splittings, namely all the admissible conventions for clock synchronization, and all the admissible non-inertial frames centered on time-like observers are *gauge equivalent*. By adding four gauge-fixing constraints $\chi^\mu(\tau, \sigma^r) = z^\mu(\tau, \sigma^r) - z^\mu_M(\tau, \sigma^r) \approx 0$ [$z^\mu_M(\tau, \sigma^r)$ being an admissible embedding], satisfying the orbit condition $det\, |\{\chi^\mu(\tau, \sigma^r), \mathcal{H}_\nu(\tau, \sigma^r_1)| \neq 0$ (implying the selection of only one point in each gauge orbit inside the constraint sub-manifold), we identify the description of the system in the associated inertial frame centered on a given time-like observer. The resulting effective Hamiltonian for the τ-evolution turns out to contain the potentials of the *relativistic inertial forces* present in the given non-inertial frame. Since a non-inertial frame means the use of its radar coordinates, we see that already in special relativity *non-inertial Hamiltonians are coordinate-dependent quantities* like the notion of energy density in general relativity.

As a consequence, the gauge variables $z^\mu(\tau, \sigma^r)$ describe the *spatio-temporal appearances* of the phenomena in non-inertial frames, which, in turn, are associated to extended physical laboratories using a metrology for their measurements compatible with the notion of simultaneity of the non-inertial frame (think to the description of the Earth given by GPS). Therefore, notwithstanding mathematics tends to use only coordinate-independent notions, physical metrology forces us to consider intrinsically coordinate-dependent quantities like the non-inertial Hamiltonians. For instance, the motion of satellites around the Earth is governed by a set of empirical coordinates contained in the software of NASA computers: this is a metrological standard of space-time around the Earth with a poorly understood connection with the purely theoretical coordinate systems. In a few years the European Space Agency will start the project ACES about the synchronization of a high-precision laser-cooled atomic clock on the space station with similar clocks on the Earth surface by means of microwave signals. If the accuracy of 5 picosec. will be achieved, it will be possible to make a coordinate-dependent test of effects at the order $1/c^3$, like the second order Sagnac effect (sensible to Earth acceleration) and the general relativistic Shapiro time-delay created by the geoid. The two-way velocity of light between an Earth station and the space station and the synchronization of the respective clocks are two faces of the same problem.

Inertial frames centered on inertial observers are a special case of gauge fixing in parametrized Minkowski theories. For each configuration of an isolated system there is an special 3+1 splitting associated to it: the foliation with space-like hyper-planes orthogonal to the conserved time-like 4-momentum of the isolated system. This identifies an intrinsic inertial frame, the *rest-frame*, centered on a suitable inertial observer (the Fokker-Pryce center of inertia of the isolated system) and allows to define the *Wigner-covariant rest-frame instant form of dynamics* for every isolated system [see Dirac (1949) for the various forms of dynamics].

This framework made possible to develop a coherent formalism for all the aspects of relativistic kinematics both for N particle systems and continuous bodies and fields [see Alba, Lusanna and Pauri (2002, 2004)]: i) the classification of the intrinsic notions of collective variables (canonical non-covariant center of mass; covariant non-canonical Fokker-Pryce center of inertia; non-covariant non-canonical Møller center of energy); ii) canonical bases of center-of-mass and relative variables; iii) canonical spin bases and dynamical body-frames for the rotational kinematics of deformable systems; iv) multipolar expansions for isolated and open systems; v) the relativistic theory of orbits; vi) the Møller radius (a classical unit of length identifying the region of non-covariance of the canonical center of mass of a spinning system around the covariant Fokker-Pryce center of inertia; it is an effect induced by the Lorentz signature of the 4-metric; it could be used as a physical ultraviolet cutoff in quantization). See Alba, Lusanna and Pauri (2005) for a comprehensive review.

Let us remark that in parametrized Minkowski theories a relativistic particle with world-line $x_i^\mu(\tau)$ is described only by the 3-coordinates $\sigma^r = \eta_i^r(\tau)$ defined by $x_i^\mu(\tau) = z^\mu(\tau, \eta_i^r(\tau))$ and by the conjugate canonical momenta $\kappa_{ir}(\tau)$. The usual 4-momentum $p_{i\mu}(\tau)$ is a derived quantity satisfying the mass-shell constraint $\epsilon\, p_i^2 = m_i^2$. Therefore, we have a different description for positive- and negative- energy particles. All the particles on an admissible surface Σ_τ are simultaneous by construction: this eliminates the problem of relative times, which for a long time has been an obstruction to the theory of relativistic bound states and to relativistic statistical mechanics.

Let us also remark that, differently from Fermi coordinates (a purely theoretical construction), radar 4-coordinates can be operationally defined. As shown in Alba and Lusanna (2005a), given four functions satisfying certain restrictions induced by the Møller conditions, the on-board computer of a spacecraft may establish a grid of radar 4-coordinates in its future.

In Alba and Lusanna (2005b) there is the quantization of relativistic scalar and spinning particles in a class of non-inertial frames, whose simultaneity surfaces Σ_τ are space-like hyper-planes with arbitrary admissible linear acceleration and carrying arbitrary admissible differentially rotating 3-coordinates. It is based on a multi-temporal quantization scheme for systems with first-class constraints, in which only the particle degrees of freedom $\eta_i^r(\tau)$, $\kappa_{ir}(\tau)$ are quantized. The gauge variables, describing the appearances (inertial effects) of the motion in non-inertial frames, are treated as c-numbers (like the time in the Schroedinger equation with a time-dependent Hamiltonian) and the physical scalar product does not depend on them. The previously quoted relativistic kinematics has made possible to separate the center of mass and to verify that the spectra of relativistic bound states in non-inertial frames are only modified by inertial effects, being obtained from the inertial ones by means of a time-dependent unitary transformation. The non-relativistic limit allows to recover the few existing attempts of quantization in non-inertial frames as particular cases.

The main open problem is the quantization of the scalar Klein-Gordon field in non-inertial frames, due to the Torre and Varadarajan (1999) no-go theorem, according to which in general the evolution from an initial space-like hyper-surface to a final one is *not unitary* in the Tomonaga-Schwinger formulation of quantum field theory. From the 3+1 point of view there is evolution only among the leaves of an admissible foliation and the possible way out from the theorem lies in the determination of all the admissible 3+1 splittings of Minkowski space-time satisfying the following requirements: i) existence of an instantaneous Fock space on each simultaneity surface Σ_τ (i.e. the Σ_τ's must admit a generalized Fourier transform); ii) unitary equivalence of the Fock spaces on Σ_{τ_1} and Σ_{τ_2} belonging to the same foliation (the associated Bogoliubov transformation must be Hilbert-Schmidt), so that the non-inertial Hamiltonian is a Hermitean operator; iii) unitary gauge equivalence of the 3+1 splittings with the Hilbert-Schmidt property. The overcoming of the no-go theorem would help also in quantum field theory in curved space-times and in condensed matter (here the non-unitarity implies non-Hermitean Hamiltonians and negative energies).

As a final comment, let us note that nearly every relevant physical system is a field theory with gauge symmetries. This means that i) we have singular Lagrangian densities whose Hessian matrix has zero eigenvalues; ii) we must use the second Noether theorem; iii) we must distinguish between gauge theories (invariance under a local Lie group acting on an inner space) and theories with spatio-temporal invariances (invariance under a group of diffeomorphisms acting also on the space-time); iv) the Hamiltonian description requires Dirac's theory of constraints and the physical degrees of freedom are the gauge invariant DO; v) in gauge theories the gauge variables are redundant variables present to enforce some kind of manifest covariance, while in theories with invariances under diffeomorphisms the gauge variables describe the appearances of phenomena; vi) the only known way to try to separate DO from gauge variables (namely to separate the elliptic partial differential equations connected with the constraints and to arrive to hyperbolic Hamilton equations for the DO with a well-posed Cauchy problem starting from a set of field equations restricted by the Noether identities) makes use of canonical transformations (the Shanmugadhasan ones defined in the next Section) in field theory.

Now most of the mathematics used in these steps is not yet rigorously defined, so that all the results hold only at a heuristic level.

III. THE CHRONOGEOMETRICAL STRUCTURE OF GENERAL RELATIVITY

In the years 1913-16 Einstein developed general relativity relying on the equivalence principle (equality of inertial and gravitational masses of bodies in free fall). It suggested him the impossibility to distinguish a constant gravitational field from the effects of a constant acceleration by means of local experiments in sufficiently small regions where the effects of tidal forces are negligible. This led to the geometrization of the gravitational interaction and to the replacement of Minkowski space-time with a pseudo-Riemannian 4-manifold M^4 with non vanishing curvature Riemann tensor. The principle of general covariance (see Norton (1993) for a review), at the basis of the tensorial nature of Einstein's equations, has the two following consequences: i) the invariance of the Hilbert action under *passive* diffeomorphisms (the coordinate transformations in M^4), so that the second Noether theorem implies the existence of first-class constraints at the Hamiltonian level; ii) the mapping of solutions of Einstein's equations among themselves under the action of *active* diffeomorphisms of M^4 extended to the tensors over M^4 (dynamical symmetries of Einstein's equations).

The basic field of metric gravity is the 4-metric tensor with components ${}^4g_{\mu\nu}(x)$ in an arbitrary coordinate system of M^4. The peculiarity of gravity is that the 4-metric field, differently from the fields of electromagnetic, weak and strong interactions and from the matter fields, has a *double role*: i) it is the mediator of the gravitational interaction (in analogy to all the other gauge fields); ii) it determines the chrono-geometric structure of the space-time M^4 in a dynamical way through the line element $ds^2 = {}^4g_{\mu\nu}(x)\, dx^\mu\, dx^\nu$. As a consequence, the gravitational field *teaches relativistic causality* to all the other fields: for instance it tells to classical rays of light and to quantum photons and gluons which are the allowed trajectories for massless particles in each point of M^4.

Let us make a comment about the two main existing approaches to the quantization of gravity.

1) Effective quantum field theory and string theory. This approach contains the standard model of elementary particles and its extensions. However, since the quantization, namely the definition of the Fock space, requires a background space-time where it is possible to define creation and annihilation operators, one must use the splitting ${}^4g_{\mu\nu} = {}^4\eta^{(B)}_{\mu\nu} + {}^4h_{\mu\nu}$ and quantize only the perturbation ${}^4h_{\mu\nu}$ of the background 4-metric $\eta^{(B)}_{\mu\nu}$ (usually B is either Minkowski or DeSitter space-time). In this way property ii) is lost (one uses the fixed non-dynamical chrono-geometrical structure of the background space-time), gravity is replaced by a field of spin two over the background (and passive diffeomorphisms are replaced by gauge transformations acting in an inner space) and the only difference among gravitons, photons and gluons lies in their quantum numbers. The main remnant of general covariance is the fact that the theory is not perturbatively renormalizable.

2) Loop quantum gravity. This approach never introduces a background space-time, but being inequivalent to a Fock space, has problems to incorporate particle physics. It uses a fixed 3+1 splitting of the space-time M^4 and it is a quantization of the associated instantaneous 3-spaces Σ_τ (quantum geometry). However, there is no known way to implement a consistent unitary evolution (the problem of the super-hamiltonian constraint) and, since it is usually formulated in spatially compact space-times without boundary, there is no notion of a Poincare' group (and therefore no extra dimensions) and a problem of time (frozen picture without evolution).

For outside points of view on loop quantum gravity and string theory see Nicolai, Peeters and Zamaklar (2005) and Smolin (2003), respectively.

Let us remark that in all known formulations particle and nuclear physics are a chapter of the theory of representations of the Poincare' group in inertial frames in the spatially non-compact Minkowski space-time. This implies for instance that to speak of nucleo-synthesis in spatially compact space-times in the cosmological context is a big extrapolation.

As a consequence, if one looks at general relativity from the point of view of particle physics, the main problem to get a unified theory is how to reconcile the Poincare' group (the kinematical group of the transformations connecting inertial frames) with the diffeomorphism group implying the non-existence of global inertial frames in general relativity (special relativity holds only in a small neighborhood of a body in free fall).

Let us consider the ADM formulation of metric gravity [Arnowitt, Deser and Misner (1962)] and its extension to tetrad gravity (needed to describe the coupling of gravity to fermions; it is a theory of time-like observers endowed with a tetrad field, whose time-like axis is the unit 4-velocity and whose spatial axes are associated to a choice of three gyroscopes) obtained by replacing the ten configurational variables ${}^4g_{\mu\nu}(x)$ with the sixteen cotetrad fields ${}^4E^{(\alpha)}_\mu(x)$ by means of the decomposition ${}^4g_{\mu\nu}(x) = {}^4E^{(\alpha)}_\mu(x)\, {}^4\eta_{(\alpha)(\beta)}\, {}^4E^{(\beta)}_\nu(x)$ [(α) are flat indices]. Then, after having restricted the model to globally hyperbolic, topologically trivial, spatially non-compact space-times (admitting a global notion of time), let us introduce a 3+1 splitting of the space-time M^4 and let choose the world-line of a time-like observer. As in special relativity, let us make a coordinate transformation to observer-dependent radar 4-coordinates, $x^\mu \mapsto \sigma^A = (\tau, \sigma^r)$, adapted to the 3+1 splitting and using the observer world-line as origin of the 3-coordinates. Again the inverse transformation, $\sigma^A \mapsto x^\mu = z^\mu(\tau, \sigma^r)$, defines the embedding of the leaves Σ_τ into M^4. These leaves Σ_τ (assumed to be Riemannian 3-manifolds diffeomorphic to R^3, so that they admit global 3-coordinates σ^r and a unique 3-geodesic joining any pair of points in Σ_τ) are both Cauchy surfaces and simultaneity surfaces corresponding to a convention for clock synchronization. For the induced 4-metric we get

$$
\begin{aligned}
{}^4g_{AB}(\sigma) &= \frac{\partial z^\mu(\sigma)}{\partial \sigma^A}\, {}^4g_{\mu\nu}(x)\, \frac{\partial z^\nu(\sigma)}{\partial \sigma^B} = \\
&= {}^4E^{(\alpha)}_A\, {}^4\eta_{(\alpha)(\beta)}\, {}^4E^{(\beta)}_B = \\
&= \epsilon \begin{pmatrix} (N^2 - {}^3g_{rs}\, N^r\, N^s) & -{}^3g_{su}\, N^u \\ -{}^3g_{ru}\, N^u & -{}^3g_{rs} \end{pmatrix} (\sigma).
\end{aligned}
$$

Here ${}^4E^{(\alpha)}_A(\tau, \sigma^r)$ are adapted cotetrad fields, $N(\tau, \sigma^r)$ and $N^r(\tau, \sigma^r)$ the lapse and shift functions and ${}^3g_{rs}(\tau, \sigma^r)$ the 3-metric on Σ_τ with signature $(+++)$. We see that in general relativity the quantities $z^\mu_A = \partial z^\mu / \partial \sigma^A$ are no more cotetrad fields on M^4 differently from what happens in special relativity: now they are only transition functions between coordinate charts, so that the dynamical fields are now the real cotetrad fields ${}^4E^{(\alpha)}_A(\tau, \sigma^r)$ and not the embeddings $z^\mu(\tau, \sigma^r)$.

Let us try to identify a class of space-times and an associated suitable family of admissible 3+1 splittings able to incorporate particle physics and giving a model for the solar system or our galaxy (and hopefully allowing an extension to the cosmological context) with the following further requirements [Lusanna (2001)]:

1) M^4 must be asymptotically flat at spatial infinity and the 4-metric must tend asymptotically at spatial infinity to the Minkowski 4-metric in every coordinate system (this implies that the 4-diffeomorphisms must tend to the identity at spatial infinity). Therefore, in these space-times there is an *asymptotic background 4-metric* and this will allow to avoid the decomposition ${}^4g_{\mu\nu} = {}^4\eta_{\mu\nu} + {}^4h_{\mu\nu}$ in the bulk.

2) The boundary conditions on each leaf Σ_τ of the admissible 3+1 splittings must be such to reduce the Spi group of asymptotic symmetries [see Wald (1984)] to the ADM Poincare' group. This means that *super-translations* (direction-dependent quasi Killing vectors, obstruction to the definition of angular momentum in general relativity) must be absent, namely that all the fields must tend to their asymptotic limits in a direction- independent way [see Regge and Teitelboim (1974) and Beig and O'Murchadha (1987)]. This is possible only if the admissible 3+1 splittings have all the leaves Σ_τ tending to Minkowski space-like hyper-planes orthogonal to the ADM 4-momentum at spatial infinity [Lusanna (2001)]. In turn this implies that every Σ_τ is the rest frame of the instantaneous 3-universe and that there are asymptotic inertial observers to be identified with the *fixed stars* (in a future extension to the cosmological context they could be identified with the privileged observers at rest with respect to the background cosmic radiation). This requirement implies that the shift functions vanish at spatial infinity [$N^r(\tau, \sigma^r) \rightarrow O(1/|\sigma|^\epsilon)$, $\epsilon > 0$, $\sigma^r = |\sigma|\, \hat{u}^r$], where the lapse function tends to 1 [$N(\tau, \sigma^r) \rightarrow 1 + O(1/|\sigma|^\epsilon)$] and the 3-metric tends to the Euclidean one [${}^3g_{rs}(\tau, \sigma^u) \rightarrow \delta_{rs} + O(1/|\sigma|)$].

3) The admissible 3+1 splittings should have the leaves Σ_τ admitting a generalized Fourier transform (namely they should be Lichnerowicz (1964) 3-manifolds with involution, so to have the possibility to define instantaneous Fock spaces in a future attempt of quantization).

4) All the fields on Σ_τ should belong to suitable weighted Sobolev spaces, so that M^4 has no Killing vectors and Yang-Mills fields on Σ_τ do not present Gribov ambiguities (due to the presence of gauge symmetries and gauge copies) [Moncrief (1979), Lusanna (1995), DePietri, Lusanna, Martucci and Russo (2002)].

In absence of matter the Christodoulou and Klainermann (1993) space-times are good candidates: they are near Minkowski space-time in a norm sense, avoid singularity theorems by relaxing the requirement of conformal completability (so that it is possible to follow solutions of Einstein's equations on long times) and admit gravitational radiation at null infinity.

Since the simultaneity leaves Σ_τ are the rest frame of the instantaneous 3-universe, at the Hamiltonian level it is possible to define the rest-frame instant form of metric and tetrad gravity [Lusanna (2001), Lusanna and Russo (2002), DePietri, Lusanna, Martucci and Russo (2002)]. If matters is present, the limit of this description for vanishing Newton constant will produce the rest-frame instant form description of the same matter in the framework of parametrized Minkowski theories and the ADM Poincare' generators will tend to the kinematical Poincare' generators of special relativity. Therefore we have obtained a model admitting *a deparametrization of general relativity to special relativity.* It is not known whether the rest-frame condition can be relaxed in general relativity without having super-translations reappearing, since the answer to this question is connected with the non-trivial problem of boosts in general relativity.

Let us now come back to ADM tetrad gravity. The time-like vector ${}^4E^A_{(o)}(\tau,\sigma^r)$ of the tetrad field ${}^4E^A_{(\alpha)}(\tau,\sigma^r)$ dual to the cotetrad field ${}^4E^{(\alpha)}_A(\tau,\sigma^r)$ may be rotated to become the unit normal to Σ_τ in each point by means of a standard Wigner boost for time-like Poincare' orbits depending on three parameters $\varphi_{(a)}(\tau,\sigma^r)$, $a=1,2,3$: ${}^4E^A_{(o)}(\tau,\sigma^r) = L^A{}_B(\varphi_{(a)}(\tau,\sigma^r))\,{}^4\check{E}^B_{(o)}(\tau,\sigma^r)$. This allows to define the following cotetrads adapted to the 3+1 splitting (the so-called *Schwinger time gauge*) ${}^4\check{E}^{(o)}_A(\tau,\sigma^r) = \Big(N(\tau,\sigma^r);0\Big)$, ${}^4\check{E}^{(a)}_A(\tau,\sigma^r) = \Big(N_{(a)}(\tau,\sigma^r); {}^3e_{(a)r}(\tau,\sigma^r)\Big)$, where ${}^3e_{(a)r}(\tau,\sigma^r)$ are cotriads fields on Σ_τ [tending to $\delta_{(a)r} + O(1/|\sigma|)$ at spatial infinity] and $N_{(a)} = N^r\,{}^3e_{(a)r}$. As a consequence, the sixteen cotetrad fields may be replaced by the fields $\varphi_{(a)}(\tau,\sigma^r)$, $N(\tau,\sigma^r)$, $N_{(a)}(\tau,\sigma^r)$, ${}^3e_{(a)r}(\tau,\sigma^r)$, whose conjugate canonical momenta will be denoted as $\pi_N(\tau,\sigma^r)$, $\pi_{\vec{N}\,(a)}(\tau,\sigma^r)$, $\pi_{\vec{\varphi}\,(a)}(\tau,\sigma^r)$, ${}^3\pi^r_{(a)}(\tau,\sigma^r)$.

The local invariances of the ADM action imply the existence of 14 first-class constraints (10 primary and 4 secondary): i) $\pi_N(\tau,\sigma^r)\approx 0$ implying the secondary super-hamiltonian constraint $\mathcal{H}(\tau,\sigma^r)\approx 0$; ii) $\pi_{\vec{N}\,(a)}(\tau,\sigma^r)\approx 0$ implying the secondary super-momentum constraints $\mathcal{H}_{(a)}(\tau,\sigma^r)\approx 0$; iii) $\pi_{\vec{\varphi}\,(a)}(\tau,\sigma^r)\approx 0$; iv) three constraints $M_{(a)}(\tau,\sigma^r)\approx 0$ generating rotations of the cotriads. As a consequence there are 14 gauge variables describing the *generalized inertial effects* in the non-inertial frame defined by the chosen admissible 3+1 splitting of M^4 centered on an arbitrary time-like observer. The remaining independent "two + two" degrees of freedom are the gauge invariant DO of the gravitational field describing *generalized tidal effects*. The same degrees of freedom emerge in ADM metric gravity, where the configuration variables N, N^r, ${}^4g_{rs}$ with conjugate momenta π_N, $\pi_{\vec{N}\,r}$, ${}^3\Pi^{rs}$, are restricted by 8 first-class constraints ($\pi_N(\tau,\sigma^r)\approx 0 \rightarrow \mathcal{H}(\tau,\sigma^r)\approx 0$, $\pi_{\vec{N}\,r}(\tau,\sigma^r)\approx 0 \rightarrow \mathcal{H}^r(\tau,\sigma^r)\approx 0$).

In the canonical approach it is possible to make a separation of the gauge variables from the DO by means of a Shanmugadhasan (1973) canonical transformation [see also Lusanna (1993)]. These transformations define a canonical basis adapted to the existing first-class constraints. The constraint presymplectic sub-manifold defined in phase space by the original first-class constraints is now defined by the vanishing of an equal number of the new momenta (Abelianization of the first-class constraints), whose conjugate configuration variables are the arbitrary gauge variables. The remaining pairs of the new canonical variables are the DO. While in finite dimensions the local existence of the Shanmugadhasan canonical transformations can be demonstrated by using Lie's theory of function groups and Levi-Civita's results about systems of equations of motion which cannot be put in normal form, in field theory the situation is more complicated, because certain constraints are elliptic partial differential equations. In function spaces where these equations do not admit zero modes, these canonical transformations are assumed to exist at least locally. Dirac (1955) used them to find the DO of the electromagnetic field: they are the transverse vector potential $\vec{A}_\perp$ and the transverse electric field $\vec{E}_\perp$ like in the radiation gauge.

Since no-one knows how to solve the super-hamiltonian constraint (except that in the post-Newtonian approximation), the best we can do is to look for a quasi-Shanmugadhasan canonical transformation adapted to the other 13 first-class constraints (the only constraints to be Abelianized are $M_{(a)}(\tau,\sigma^r) \approx 0$ and $\mathcal{H}_{(a)}(\tau,\sigma^r) \approx 0$) [DePietri, Lusanna, Martucci and Russo (2002)]:

$$
\begin{array}{|c|c|c|c|}
\hline
\varphi^{(a)} & N & N_r & {}^3e_{(a)r} \\
\hline
\approx 0 & \approx 0 & \approx 0 & {}^3\tilde{\pi}^r_{(a)} \\
\hline
\end{array}
\quad \longrightarrow \quad
\begin{array}{|ccccc|c|c|}
\hline
\varphi^{(a)} & N & N_{(a)} & \alpha_{(a)} & \xi^r & \phi & r_{\bar a} \\
\hline
\approx 0 & \approx 0 & \approx 0 & \approx 0 & \approx 0 & \pi_\phi & \pi_{\bar a} \\
\hline
\end{array}
$$

Here, $\alpha_{(a)}(\tau,\sigma^r)$ are three Euler angles and $\xi^r(\tau,\sigma^r)$ are three parameters giving a coordinatization of the action of 3-diffeomorphisms on the cotriads ${}^3e_{(a)r}(\tau,\sigma^r)$. The configuration variable $\phi(\tau,\sigma^r) = \Big(det\,{}^3g(\tau,\sigma^r)\Big)^{1/12}$ is the conformal factor of the 3-metric: it can be shown that it is the unknown in the super-hamiltonian constraint (also named the Lichnerowicz equation). The gauge variables are N, $N_{(a)}$, $\varphi_{(a)}$, $\alpha_{(a)}$, ξ^r and π_ϕ, while $r_{\bar a}$, $\pi_{\bar a}$, $\bar a = 1,2$, are the DO of the gravitational field (in general they are not tensorial quantities).

Even if we do not know the expression of the final variables in terms of the original ones, we note that this a *point* canonical transformation with known inverse

$$
{}^3e_{(a)r}(\tau,\sigma^u) = {}^3R_{(a)(b)}(\alpha_{(e)}(\tau,\sigma^u))\,\frac{\partial \xi^s(\tau,\sigma^u)}{\partial \sigma^r}\,\phi^2(\tau,\vec{\xi}(\tau,\sigma^u))\,{}^3\hat{e}_{(b)s}(r_{\bar a}(\tau,\xi^u(\tau,\sigma^v))\,),
$$

as implied by the study of the gauge transformations generated by the first-class constraints [${}^3\hat{e}_{(a)r}$ are reduced cotriads, which depend only on the two configurational DO $r_{\bar a}$].

The point nature of the canonical transformation implies that the old cotriad momenta are linear functionals of the new momenta. The kernel connecting the old and new momenta satisfy elliptic partial differential equations implied by i) the canonicity conditions; ii) the super-momentum constraints $\mathcal{H}_{(a)}(\tau,\sigma^r) \approx 0$; iii) the rotation constraints $M_{(a)}(\tau,\sigma^r) \approx 0$.

The first-class constraints are the generators of the Hamiltonian gauge transformations, under which the ADM action is quasi-invariant (second Noether theorem):

i) The gauge transformations generated by the four primary constraints $\pi_N(\tau, \sigma^r) \approx 0$, $\pi_{\vec{N}\,(a)}(\tau, \sigma^r) \approx 0$, modify the lapse and shift functions, namely how densely the simultaneity surfaces are packed in M^4 and which points have the same 3-coordinates on each Σ_τ.

ii) Those generated by the three super-momentum constraints $\mathcal{H}_{(a)}(\tau, \sigma^r) \approx 0$ change the 3-coordinates on Σ_τ.

iii) Those generated by the super-hamiltonian constraint $\mathcal{H}(\tau, \sigma^r) \approx 0$ transform an admissible 3+1 splitting into another admissible one by realizing a normal deformation of the simultaneity surfaces Σ_τ [see Teitelboim (1980)]. As a consequence, all the conventions about clock synchronization are gauge equivalent as in special relativity.

iv) Those generated by $\pi_{\vec{\varphi}\,(a)}(\tau, \sigma^r) \approx 0$, $M_{(a)}(\tau, \sigma^r) \approx 0$, change the cotetrad fields with local Lorentz transformations.

In the rest-frame instant form of tetrad gravity there are the three extra first-class constraints $P^r_{ADM} \approx 0$ (vanishing of the ADM 3-momentum as rest-frame conditions). They generated gauge transformations which change the time-like observer whose world-line is used as origin of the 3-coordinates.

Finally let us see which is the Dirac Hamiltonian H_D generating the τ-evolution in ADM canonical gravity. In spatially compact space-times without boundary H_D is a linear combination of the primary constraints (each one multiplied by an arbitrary Dirac multiplier, the Hamiltonian version of the undetermined velocities of the configurational approach whose existence is implied by the second Noether theorem) plus the secondary super-hamiltonian and super-momentum constraints multiplied by the lapse and shift functions respectively (consequence of the Legendre transform). As a consequence, $H_D \approx 0$ and in the reduced phase space (quotient of the constraint sub-manifold with respect to the group of gauge transformations) we get a vanishing Hamiltonian. This implies the so-called *frozen picture* and the problem of how to reintroduce a temporal evolution. Usually one considers the normal (time-like) deformation of Σ_τ induced by the super-hamiltonian constraint as an evolution in a local time variable to be identified (the multi-fingered time point of view with a local either extrinsic or intrinsic time): this is the so-called *Wheeler-DeWitt interpretation* (Kuchar (1992,1993) says that the super-hamiltonian constraint must not be interpreted as a generator of gauge transformations, but as an effective Hamiltonian).

On the contrary, in spatially non-compact space-times the definition of functional derivatives and the existence of a well-posed Hamiltonian action principle (with the possibility of a good control of the surface terms coming from integration by parts) require the addition of the *DeWitt (1967) surface term* (living on the surface at spatial infinity) to the Hamiltonian. It can be shown [Lusanna (2001)] that in the rest-frame instant form this term, together with a surface term coming from the Legendre transformation of the ADM action, leads to the Dirac Hamiltonian

$$H_D = \check{E}_{ADM} + (constraints) = E_{ADM} + (constraints) \approx E_{ADM}.$$

Here $\check{E}_{ADM}$ is the *strong ADM energy*, a surface term analogous to the one defining the electric charge as the flux of the electric field through the surface at spatial infinity in electromagnetism. Since we have $\check{E}_{ADM} = E_{ADM} + (constraints)$, we see that the non-vanishing part of the Dirac Hamiltonian is the *weak ADM energy* $E_{ADM} = \int d^3\sigma\, \mathcal{E}_{ADM}(\tau, \sigma^r)$, namely the integral over Σ_τ of the ADM energy density (in electromagnetism this corresponds to the definition of the electric charge as the volume integral of matter charge density). Therefore there is no frozen picture but a consistent τ-evolution.

However, the ADM energy density $\mathcal{E}_{ADM}(\tau,\sigma^r)$ is a coordinate-dependent quantity because it depends on the gauge variables (namely on the inertial effects present in the non-inertial frame): this is the *problem of energy* in general relativity. Let us remark that in most coordinate systems $\mathcal{E}_{ADM}(\tau,\sigma^r)$ does not agree with the pseudo-energy density defined in terms of the Landau-Lifschiz pseudo-tensor.

As a consequence, to get a deterministic evolution for the DO we must fix the gauge completely, that is we have to add 14 gauge-fixing constraints satisfying an orbit condition (so that only one point in each gauge orbit inside the constraint sub-manifold is selected) and to pass to Dirac brackets (the symplectic structure of the selected copy of the reduced phase space). The correct way to do it in constraint theory, when there are secondary constraints, is the following one:

i) Add a gauge-fixing constraint to the secondary super-hamiltonian constraint (the choice $\pi_\phi(\tau,\sigma^r)\approx 0$ implies that the DO $r_{\bar a}$, $\pi_{\bar a}$, remain canonical even if we do not know how to solve this constraint). This gauge-fixing fixes the form of Σ_τ, i.e. the convention for the synchronization of clocks. The τ-constancy of this gauge-fixing constraint (needed for consistency) generates a gauge-fixing constraint to the primary constraint $\pi_N(\tau,\sigma^r)\approx 0$ for the determination of the lapse function. The τ-constancy of this new gauge fixing determines the Dirac multiplier in front of the primary constraint.
ii) Add three gauge-fixings to the secondary super-momentum constraints $\mathcal{H}_{(a)}(\tau,\sigma^r)\approx 0$. This fixes the 3-coordinates on each Σ_τ. The τ-constancy of these gauge fixings generates the three gauge fixings to the primary constraints $\pi_{\vec N\,(a)}(\tau,\sigma^r)\approx 0$ and leads to the determination of the shift functions (i.e. of the appearances of gravito-magnetism). The τ-constancy of these new gauge fixings determines the Dirac multipliers in front of the three primary constraints.
iii) Add six gauge-fixing constraints to the primary constraints $\pi_{\vec\varphi\,(a)}(\tau,\sigma^r)\approx 0$, $M_{(a)}(\tau,\sigma^r)\approx 0$. This is a fixation of the cotetrad field which includes a convention on the choice of the three gyroscopes of every time-like observer of the two congruences associated to the chosen 3+1 splitting of M^4. Their τ-constancy determines the six Dirac multipliers in front of these primary constraints.
iv) In the rest-frame instant form we must also add three gauge fixings to the rest-frame conditions $P^r_{ADM}\approx 0$. The natural ones are obtained with the requirement that the three ADM boosts vanish. In this way we select a special time-like observer as origin of the 3-coordinates (like the Fokker-Pryce center of inertia in special relativity).

In this way all the gauge variables are fixed to be either numerical functions or well determined functions of the DO. As a consequence, in a completely fixed gauge (i.e. in a non-inertial frame centered on a time-like observer and with its pattern of inertial forces, corresponding to an extended physical laboratory with fixed metrological conventions) the ADM energy density $\mathcal{E}_{ADM}(\tau,\sigma^r)$ becomes a well defined function only of the DO and the Hamilton equations for them with E_{ADM} as Hamiltonian are a hyperbolic system of partial differential equations for their determination. For each choice of Cauchy data for the DO on a Σ_τ, we obtain a solution of Einstein's equations in the radar 4-coordinate system associated to the chosen 3+1 splitting of M^4.

A universe M^4 (a 4-geometry) is the equivalence class of all the completely fixed gauges with gauge equivalent Cauchy data for the DO on the associated Cauchy and simultaneity surfaces Σ_τ. In each gauge we find the solution for the DO in that gauge (the tidal effects) and then the explicit form of the gauge variables (the inertial effects). Moreover, also the extrinsic curvature of the simultaneity surfaces Σ_τ is determined. Since the simultaneity surfaces are asymptotically flat, it is possible to determine their embeddings $z^\mu(\tau, \sigma^r)$ in M^4. As a consequence, differently from special relativity, the conventions for clock synchronization and the whole chrono-geometrical structure of M^4 (gravito-magnetism, 3-geodesic spatial distance on Σ_τ, trajectories of light rays in each point of M^4, one-way velocity of light) are *dynamically determined* .

Let us remark that, if we look at Minkowski space-time as a special solution of Einstein's equations with $r_{\bar{a}}(\tau, \sigma^r) = \pi_{\bar{a}}(\tau, \sigma^r) = 0$ (zero Riemann tensor, no tidal effects, only inertial effects), we find [Lusanna (2001)] that the dynamically admissible 3+1 splittings (non-inertial frames) must have the simultaneity surfaces Σ_τ *3-conformally flat*, because the conditions $r_{\bar{a}}(\tau, \sigma^r) = \pi_{\bar{a}}(\tau, \sigma^r) = 0$ imply the vanishing of the Cotton-York tensor of Σ_τ. Instead, in special relativity, considered as an autonomous theory, all the non-inertial frames compatible with the Møller conditions are admissible, namely there is much more freedom in the conventions for clock synchronization.

A first application of this formalism [Agresti, DePietri, Lusanna and Martucci (2004)]] has been the determination of post-Minkowskian background-independent gravitational waves in a completely fixed non-harmonic 3-orthogonal gauge with diagonal 3-metric. It can be shown that the requirements $r_{\bar{a}}(\tau, \sigma^r) << 1$, $\pi_{\bar{a}}(\tau, \sigma^r) << 1$ lead to a weak field approximation based on a Hamiltonian linearization scheme: i) linearize the Lichnerowicz equation, determine the conformal factor of the 3-metric and then the lapse and shift functions; ii) find E_{ADM} in this gauge and disregard all the terms more than quadratic in the DO; iii) solve the Hamilton equations for the DO. In this way we get a solution of linearized Einstein's equations, in which the configurational DO $r_{\bar{a}}(\tau, \sigma^r)$ play the role of the two polarizations of the gravitational wave and we can evaluate the embedding $z^\mu(\tau, \sigma^r)$ of the simultaneity surfaces of this gauge explicitly.

IV. EINSTEIN'S HOLE ARGUMENT

In 1914 Einstein (1914), during his researches for developing general relativity, faced the problem arising from the fact that the requirement of general covariance would involve a threat to the physical objectivity of the points of space-time M^4, which in classical field theories are usually assumed to have a well defined individuality. He formulated the Hole Argument and stated (our *emphasis*)

> That this *requirement of general covariance*, which *takes away from space and time the last remnant of physical objectivity*, is a natural one, will be seen from the following reflexion... (Einstein, 1916, p.117)

Assume that M^4 contains a *hole* $\mathcal{H}$, that is an open region where all the non-gravitational fields vanish. It is implicitly assumed that the Cauchy surface for Einstein's equations lies outside $\mathcal{H}$. Let us consider an active diffeomorphism A which re-maps the points inside $\mathcal{H}$, but is the identity outside $\mathcal{H}$. For any point $x \in \mathcal{H}$ we have $x \mapsto D_A\, x \in \mathcal{H}$. The induced active diffeomorphism on the 4-metric tensor ${}^4g$, solution of Einstein's equations, will map it into another solution $D^*_A\, {}^4g$ (D^*_A is a dynamical symmetry of Einstein's equations) defined by $D^*_A\,{}^4g(D_A\,x) = {}^4g(x) \neq D^*_A\,{}^4g(x)$. As a consequence, we get two solutions of Einstein's equations with the same Cauchy data outside $\mathcal{H}$ and it is not clear how to save the identification of the mathematical points of M^4.

Einstein avoided the problem with the pragmatic *point-coincidence argument*: the only real world-occurrences are the (coordinate-independent) space-time coincidences (like the intersection of two world-lines). However, the problem was reopened by Stachel (1980) and then by Earman and Norton (1987) and this opened a rich philosophical debate that is still alive today.

If we insist on the reality of space-time mathematical points independently from the presence of any physical field (the *substantivalist* point of view in philosophy of science), we are in trouble with predictability. If we say that ${}^4g$ and $D^*_A\,{}^4g$ describe the same universe (the so-called *Leibniz equivalence*), we loose any physical objectivity of the space-time points (the *relationist* point of view). Stachel (1980) suggested that a physical individuation of the point-events of M^4 could be done only by using *four individuating fields depending on the 4-metric on* M^4, namely that a tensor field on M^4 is needed to identify the points of M^4.

On the other hand, *coordinatization* is the only way to individuate the points *mathematically* since, as stressed by Hermann Weyl: "There is no distinguishing objective property by which one could tell apart one point from all others in a homogeneous space: at this level, fixation of a point is possible only by a *demonstrative act* as indicated by terms like *this* and *there*." (Weyl. 1946, p. 13).

To clarify the situation let us remember that Bergmann and Komar (1972) gave a passive re-interpretation of active diffeomorphisms as metric-dependent coordinate transformations $x^\mu \mapsto y^\mu(x, {}^4g(x))$ restricted to the solutions of Einstein's equations (i.e. *on-shell*). It can be shown that on-shell ordinary passive diffeomorphisms and the on-shell Legendre pull-back of Hamiltonian gauge transformations are two (overlapping) dense subsets of this set of on-shell metric-dependent coordinate transformations. Since the Cauchy surface for the Hole Argument lies outside the hole (where the active diffeomorphism is the identity), it follows that the passive re-interpretation of the active diffeomorphism D^*_A must be an on-shell Hamiltonian gauge transformation, so that Leibniz equivalence is identified with gauge equivalence in the sense of Dirac constraint theory (${}^4g$ and $D^*_A\,{}^4g$ belong to the same gauge orbit).

What remains to be done is to implement Stachel's suggestion according to which the *intrinsic pseudo-coordinates* of Bergmann and Komar (1960) [see also Bergmann (1962) and Komar (1958)] should be used as individuating fields. These pseudo-coordinates for M^4 (at least when there are no Killing vectors) are four scalar functions $F^A[w_\lambda]$, $A, \lambda = 1, .., 4$, of the four eigenvalues $w_\lambda({}^4g, \partial\,{}^4g)$ of the Weyl tensor. Since these eigenvalues can be shown to be in general functions of the 3-metric, of its conjugate canonical momentum (namely of the extrinsic curvature of Σ_τ) and of the lapse and shift functions, the pseudo-coordinates are well defined in phase space and can be used as a label for the points of M^4.

The final step [see Lusanna and Pauri (2005, 2004a,b)] is to implement the individuation of point-events by considering an arbitrary admissible 3+1 splitting of M^4 with a given time-like observer and the associated radar 4-coordinates σ^A and by imposing the following gauge fixings to the secondary super-hamiltonian and super-momentum constraints (the only restriction on the functions F^A is the orbit condition)

$$\chi^A(\tau, \sigma^r) = \sigma^A - F^A[w_\lambda] \approx 0.$$

In this way we break completely general covariance and we determine the gauge variables ξ^r and π_ϕ. Then the τ-constancy of these gauge fixings will produce the gauge fixings determining the lapse and shift functions. After having fixed the Lorentz gauge freedom of the cotetrads, we arrive at a completely fixed gauge in which, after the transition to Dirac brackets, we get $\sigma^A \equiv \tilde{F}^A[r_{\bar{a}}(\sigma), \pi_{\bar{a}}(\sigma)]$, namely that the radar 4-coordinates of a point in M^4_{3+1}, the copy of M^4 coordinatized with the chosen non-inertial frame, are determined *off-shell* by the four DO of that gauge: in other words the individuating fields are the genuine tidal effects of the gravitational field. By varying the functions F^A we can make an analogous off-shell identification in every other admissible non-inertial frame. The procedure is consistent, because the DO know the whole 3+1 splitting M^4_{3+1} of M^4, being functionals not only of the 3-metric on Σ_τ, but also of its extrinsic curvature.

Some consequences of this identification of the point-events of M^4 are:

1) The space-time M^4 and the gravitational field are essentially the same entity. The presence of matter modifies the solutions of Einstein equations, i.e. M^4, but does not play any role in this identification. Instead matter is fundamental for establishing a (still lacking) dynamical theory of measurement not using test objects. As a consequence, instead of the dichotomy substantivalism/relationism, we believe that this analysis - as a case study limited to the class of space-times dealt with - may offer a new more articulated point of view, which can be named *point structuralism* [see also Dorato and Pauri (2004)]. Let us recall that, in remarkable diversity with respect to the traditional historical presentation of Newton's absolutism *vis á vis* Leibniz's relationism, Newton had a much deeper understanding of the nature of space and time. In two well-known passages of *De Gravitatione*, Newton expounds what could be defined an original *proto-structuralist view* of space-time. He writes (our *emphasis*):

> Perhaps now it is maybe expected that I should define extension as substance or accident or else nothing at all. But by no means, for it has *its own manner of existence* which fits neither substance nor accidents [...] The parts of space derive their character from their positions, so that if any two could change their positions, they would change their character at the same time and each would be converted numerically into the other *qua* individuals. The parts of duration and space are only understood to be the same as they really are because of their mutual order and positions (*propter solum ordinem et positiones inter se*); nor do they have any other *principle of individuation* besides this order and position which consequently cannot be altered. (Hall & Hall, 1962, p.99, p.103.)

2) The reduced phase space of this model of general relativity is the space of abstract DO (pure tidal effects without inertial effects), which can be thought as four fields on an abstract space-time $\tilde{M}^4 = \{equivalence\, class\, of\, all\, the\, admissible\, non-inertial\, frames\, M^4_{3+1}\, containing\, the\, associated\, inertial\, effects\}$.

3) Each radar 4-coordinate system of an admissible non-inertial frame M^4_{3+1} has an associated *non-commutative structure*, determined by the Dirac brackets of the functions $\tilde{F}^A[r_{\bar{a}}(\sigma), \pi_{\bar{a}}(\sigma)]$ determining the gauge.

4) Conjecture: there should exist privileged Shanmugadhasan canonical bases of phase space, in which the DO (the tidal effects) are also *Bergmann observables*, namely coordinate-independent scalar tidal effects [see Bergmann (1961)].

As a final remark, let us note that these results on the identification of point-events are *model dependent*. In spatially compact space-times without boundary, the DO are *constants of the motion* due to the frozen picture. As a consequence, the gauge fixings $\chi^A(\tau, \sigma^r) \approx 0$ (in particular χ^τ) cannot be used to rebuild the temporal dimension: probably only the instantaneous 3-space of a 3+1 splitting can be individuated in this way.

V. OPEN PROBLEMS OF CANONICAL GRAVITY

I will finish with a list of the open problems in canonical metric and tetrad gravity for which there is a concrete hope to be clarified and solved in the near future.

i) Find a refined Shanmugadhasan canonical transformation allowing the addition of any kind of matter to the rest-frame instant form of tetrad gravity. This would allow to study the weak-field approximation to the two-body problem in a post-Minkowskian background-independent way by using a Grassmann regularization of the self-energies, following the track of Crater and Lusanna (2001) and Alba, Crater and Lusanna (2001). In these papers the use of Grassmann-valued electric charges to regularize the Coulomb self-energies allowed to arrive to the Darwin and Salpeter potentials starting from classical electrodynamics of scalar and spinning particles, instead of deriving them from quantum field theory. The solution of the Lichnerowicz equation would allow to find the expression of the relativistic Newton and gravito-magnetic action-at-a-distance potentials between the two bodies (sources, among other effects, of the Newtonian tidal effects) and the coupling of the particles to the DO of the gravitational field (the genuine tidal effects) in various radar coordinate systems: it would amount to a re-summation of the $1/c$ expansions of the Post-Newtonian approximation. Also the relativistic version of the quadrupole formula for the emission of gravitational waves from the binary system could be obtained and some understanding of how is distributed the gravitational energy in different coordinate systems could be obtained. It would also be possible to study the deviations induced by Einstein's theory from the Keplerian standards for problems like the radiation curves of galaxies, whose Keplerian interpretation implies the existence of dark matter. Finally one could try to define a relativistic gravitational micro-canonical ensemble generalizing the Newtonian one developed by Votyakov, Hidmi, De Martino and Gross (2002).

ii) With more general types of matter (relativistic fluids, electromagnetic field) it should be possible to develop Hamiltonian numerical gravity based on the Shanmugadhasan canonical basis and to study post-Minkowskian approximations based on power expansions in Newton constant. Moreover one should look for strong-field approximations to be used in the gravitational collapse of a ball of fluid.

iii) Find the Hamiltonian formulation of the Newman-Penrose formalism [see Stewart (1993)], in particular of the 10 Weyl scalars. Look for the Bergmann observables (the scalar tidal effects) and try to understand which inertial effects may have a coordinate-independent form and which are intrinsically coordinate-dependent like the ADM energy density. Look for the existence of a closed Poisson algebra of scalars and for Shanmugadhasan canonical bases incorporating the Bergmann observables, to be used to find new expressions for the super-hamiltonian and super-momentum constraints, hopefully easier to be solved.

iv) Find all the admissible 3+1 splittings of Minkowski space-time which avoid the Torre-Varadarajan no-go theorem. Then adapt these 3+1 splittings to tetrad gravity and try to see whether it is possible to arrive at a multi-temporal background- and coordinate- independent quantization of the gravitational field, in which only the Bergmann observables (the scalar tidal effects) are quantized.

v) Try to find the relativistic version of Bell inequalities by using relativistic particle quantum mechanics in non-inertial frames.

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